American University of Beirut

MATH 201

Calculus and Analytic Geometry III

Fall 2008-2009

Final Exam

Exercise 1 a. *(5 points)* If $f(u,v,w)$ is a differentiable function and if $u = x - y$, $v = y - z$, and $w = z - x$, show that $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = 0$

b. *(10 points)* Use the method of Lagrange multipliers to find the maximum and minimum values of $f(x,y) = 3x - y + 6$ on the circle $x^2 + y^2 = 4$

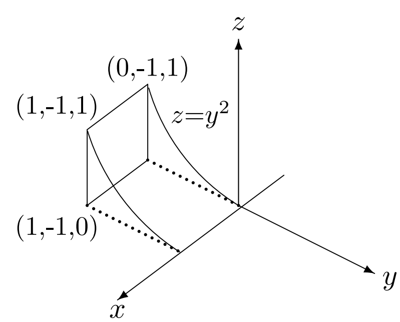
Exercise 2 *(10 points)* Convert to polar coordinates, then evaluate the following integral

$$\int_0^2 \int_{-\sqrt{1-(y-1)^2}}^0 xy^2 dxdy$$

Exercise 3 *(12 points)* Here is the region of integration of the integral

$$\int_0^1 \int_{-1}^0 \int_0^{y^2} dz\ dy\ dx$$

Rewrite the integral as an equivalent iterated integral in the other 5 orders, then evaluate one of them



Exercise 4 Let V be the volume of the region D that is bounded by the paraboloid $z = x^2 + y^2$, and the plane $z = 2y$.

a) *(8 points)* Express V as an iterated triple integral in cartesian coordinates in the order *$dz\ dx\ dy$ (do not evaluate the integral).*

b) *(10 points)* Express V as an iterated triple integral in cylindrical coordinates, then evaluate the resulting integral.

(you may use the result: $\int \sin^4 x\ dx = -\frac{\sin^3 x \cos x}{4} - \frac{3\cos x \sin x}{8} + \frac{3x}{8}$*)*

Exercise 5 Let V be the volume of the region D that is bounded below by the xy-plane, above by the sphere $x^2 + y^2 + z^2 = 4$, and on the sides by the cylinder $x^2 + y^2 = 1$.

a) *(7 points)* Express V as an iterated triple integral in spherical coordinates in the order *$d\rho\ d\phi\ d\theta$ (do not evaluate the integral).*

b) *(8 points)* Express V as an iterated triple integral in spherical coordinates in the order *$d\phi\ d\rho\ d\theta$ (do not evaluate the integral).*

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Exercise 6 **a.** *(6 points)* Find the work done by the force $F = x\mathbf{i} + y^2\mathbf{j} + (y - z)\mathbf{k}$ along the straight line from $(0, 0, 0)$ to $(1, 1, 1)$.

b. *(8 points)* Evaluate

$$\int_{(0,0,1)}^{(1,\pi/2,e)} (\ln z + e^x \sin y)dx + e^x \cos y dy + (x/z - z)dz$$

c. Find the *outward flux* of the field $F = (y - 2x)\mathbf{i} + (x + y)\mathbf{j}$ across the curve C in the first quadrant, bounded by the lines $y = 0$, $y = x$ and $x + y = 1$.

i. *(10 points)* by direct calculation

ii. *(6 points)* by Green's theorem

good luck